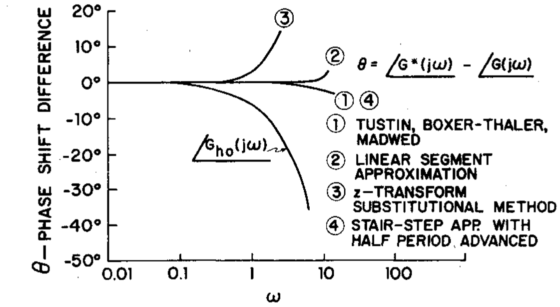

Fig. 3 Phase difference due to discretization for several methods (sampling frequency equals 16π) and phase shift for zero-order hold.

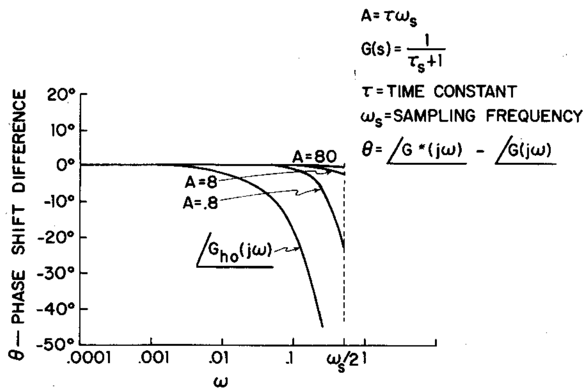

Fig. 4 Phase difference for difference sampling conditions and phase shift of zero-order hold.

Conclusions

From Fig. 3 it can be observed that a zero-order hold device introduces significantly more phase shift than all the methods considered for a simple transfer function. Similar results should prevail for complicated transfer functions. It is concluded that, in situations where a zero-order hold (i.e., D/A converter) must follow a computer, the method of discretization of a continuous system is not a major factor if phase shift is important. This is true because the sampling frequency must be relatively high to keep the phase shift of the hold device within acceptable limits, and this results in relatively small attenuation and phase shift due to discretization. For reasons of simplicity a logical choice is to use the Tustin method.

Acknowledgment

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References

[1] Beale, G.O. and Cook, G., "Frequency Domain Synthesis of Discrete Representations," IEEE Vol. IECI-23, No. 4, Nov. 1976, pp. 438-443.

[2] Rosko, J.S., *Digital Simulation of Physical Systems*, Addison Wesley, Reading, Mass., 1972.

[3] Oppenhiem, A.V. and Schafer, R.W., *Digital Signal Processing*, Prentice-Hall, Englewood Cliffs, N.J., 1975.

[4] Kuo, B.C., Singh, G., and Yackel, R., "Digital Approximation of Continuous-Data Control Systems by Point-by-Point State Comparison," *Computers and Electrical Engineering*, Vol. 1, 1973, pp. 155-170.

[5] Tustin, A., "A Method of Analyzing the Behavior of Linear Systems in Terms of Time Series," *Journal of the IEE*, Vol. 94, II-A, May 1947.

[6] Sage, A.P. and Smith, S.L., "Real-Time Digital Simulation for Systems Control," *Proceedings of the IEEE*, Vol. 54, Dec. 1966, pp. 1802-1812.

[7] Parrish, E.A., McVey, E.S., and Cook, G., "The Investigation of Optimal Discrete Approximations for Real-Time Flight Simulations," NASA TR EG-4041-102-76, March 1976.

Velocity Required for Intercept with Perturbations

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Introduction

A KNOWLEDGE of the velocity required to transfer a space vehicle in a J2 perturbing gravitational field from one position (ballistic missile or space vehicle) to another (target vector or target vehicle) is central in guidance and targeting theory. The space vehicle's thrust steering and cutoff equations generally depend on the velocity required and the instantaneous state of the vehicle obtained from the navigation system.

An analytic solution for v_{req} in a J2 field is presented[1]; however, the method could be applied in the case of arbitrary perturbations if these perturbations were known in terms of the variations of the classical elements. The theory was programmed and numerical results in terms of target miss are presented for several test cases.

The Closed-Form f and g Expressions

The fundamental expression employed in calculating the velocity required is the closed-form f and g vector equation for Keplerian motion. The equation development which follows is valid for all elliptic arcs.

$$r_T = fr + gv \tag{1}$$

where $\boldsymbol{r}_T$ is the target position in ECI (Earth-centered inertial coordinates), $\boldsymbol{r}$ is the initial position in ECI, $\boldsymbol{v}$ is the initial velocity in ECI, f, g are scaling coefficients.

$\delta\nu$ is the transfer true anomaly change:

$$f = 1 - \frac{r_T}{a(1-e^2)}(1-\cos\delta\nu) \tag{2a}$$

$$g = \frac{rr_T}{\sqrt{\mu a(1-e^2)}}\sin\delta\nu \tag{2b}$$

If no perturbations were present and the motion were Keplerian, the velocity required could be solved simply by inverting Eq. (1),

$$v_{\text{req}} \triangleq v = g^{-1}(r_T - fr) \tag{3}$$

However, J2 perturbations are assumed and therefore the focus is on specifying an auxiliary plane, here called the pseudo plane, and the pseudo instantaneous vehicle position which will allow use of the Keplerian equation (3). Equations are derived connecting the initial state osculating orbital quantities and the initial state pseudo orbital quantities.

Velocity Required in Terms of Pseudo Quantities

The plane of the final motion (r_T, v_T) is not, in general, the same as the osculating plane of the initial motion ($\boldsymbol{r}$, $\boldsymbol{v}$) and to use Keplerian analysis, a choice of plane must be made. In this development the pseudo plane is defined to be the plane of the final motion (r_T, v_T) and a pseudo initial position vector is specified, $\tilde{r}$, which lies in the pseudo plane. $\tilde{r}$ is

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defined to be that vector which causes Eq. (3) to be consistent when using pseudo-orbital quantities.

Assume we have the osculating Keplerian elements and the variation of those elements due to J2 perturbation. The first step in obtaining $\tilde{r}$ is by rotating r into the orbit plane's perifocal coordinate system using the standard rotation matrix.

$$r_{\text{(orbit plane)}} = R^T(\theta)\, r_{\text{ECI (unperturbed)}} \tag{4}$$

where

$$R(\theta) = \begin{bmatrix} c\Omega c\omega - s\Omega s\omega ci & -c\Omega s\omega - s\Omega c\omega ci & s\Omega si \\ s\Omega c\omega + c\Omega s\omega ci & -s\Omega s\omega + c\Omega c\omega ci & -c\Omega si \\ s\omega si & c\omega si & ci \end{bmatrix} \tag{5}$$

Then $r_{\text{(orbit plane)}}$ is rotated back into the ECI coordinate system, including the orientation angle perturbations

$$r'_{\text{ECI(perturbed)}} = R(\theta+\Delta\theta)\, r_{\text{(orbit plane)}} \tag{6}$$

$$= R(\theta+\Delta\theta)\,[R^T(\theta)\, r_{\text{ECI(unperturbed)}}] \tag{7}$$

Define

$$\mathcal{R}(\Delta\theta) \triangleq R(\theta+\Delta\theta)\, R^T(\theta) \tag{8}$$

Then

$$r'_{\text{ECI(perturbed)}} = \mathcal{R}(\Delta\theta)\, r_{\text{ECI(unperturbed)}} \tag{9}$$

r' does not qualify as the pseudo initial position, $\tilde{r}$, for two reasons: 1) an in-plane rotation adjustment is required and 2) its length should then be scaled via the polar equation to achieve consistency with the pseudo-orbital quantities (see Fig. 1). The in-plane rotation $\Delta\nu$ accounts for the lack of alignment of the actual and pseudo perigee vectors.

$$\Delta\nu = \nu_{r_{T(\tilde{a},\tilde{e})}} - \nu_{r_{T(a,e)}} \tag{10}$$

where the tilde quantities are associated with the pseudo orbit,

$$\tilde{a} = a + \Delta a \tag{11}$$

$$\tilde{e} = e + \Delta e \tag{12}$$

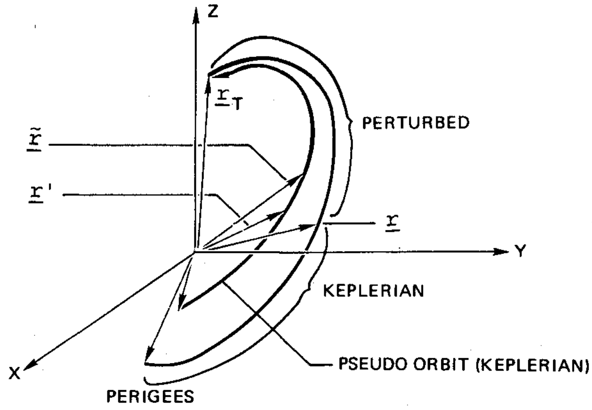

Fig. 1 Actual and pseudo initial positions and trajectories.

To improve accuracy where the initial position and target latitudes were not equal, a simple rotation bias was determined by assuming the equation form and adjusting the constants until the miss from the simulation program was minimized. The small miss associated with launch latitudes of 40 and 45 deg in Table 1 indicates the bias validity over a significant range. This bias, which was developed for an initial position latitude of 45 deg, is:

$$\Delta\nu_{\text{bias}} = -4.3\times10^{-6}\,|\text{Lat}_{r_T} - \text{Lat}_r| - 2.0\times10^{-7}\,|180\text{ deg} - |\text{Long}_{r_T} - \text{Long}_r|| \tag{13}$$

with $\Delta\nu_{\text{bias}}$ in radians and the latitudes and longitudes in degrees. The complete rotation of r to the $\tilde{r}$ direction is:

$$r''_{\text{ECI(perturbed)}} = \mathcal{R}\begin{bmatrix} \Delta i \\ \Delta\Omega \\ \Delta\omega + \Delta\nu + \Delta\nu_{\text{bias}} \end{bmatrix} r_{\text{ECI(unperturbed)}} \tag{14}$$

With the pseudo initial position oriented, scaling to the proper magnitude is straightforward via the polar equation:

$$\tilde{r} = \frac{r''}{|r''|}\cdot\frac{\tilde{a}(1-\tilde{e}^2)}{1+\tilde{e}\cos\tilde{\nu}} \tag{15}$$

Table 1 Target miss as a function of initial and target position

Case	Initial position[a], deg Lat	Long[c]	Target position[b], deg Lat	Long[c]	Miss (minimum distance from trajectory to target vector, ft)	Range, naut. miles
1	45	30	55	180	21	4617
2			55	210	18	4808
3			55	255	30	4387
4			50	180	139	4898
5			50	210	117	5109
6			50	255	136	4644
7			45	180	235	5206
8			45	210	223	5409
9			45	255	270	4903
10			40	180	37	5460
11			40	210	57	5740
12			40	255	42	5163
13	40	30	55	180	96	4905
14			50	180	41	5182
15			45	180	144	5459
16			40	180	199	5765

[a] Magnitude of initial position vector is 21,000,000 ft.
[b] Magnitude of target vector is 20,926,000 ft.
[c] Longitude positive west.

where

$$\tilde{\nu} = \tilde{\nu}_{r_T} - \text{angle}\,(r'', r_T) \tag{16}$$

with $\tilde{\nu}_{r_T}$ obtained by inverting

$$r_T = \frac{\tilde{a}(1-\tilde{e}^2)}{1+\tilde{e}\cos\tilde{\nu}_{r_T}} \tag{17}$$

Because the pseudo-orbit is Keplerian, the pseudo speed at $\tilde{r}$ is found with the *vis-viva* equation

$$\tilde{s} = [\mu(2/\tilde{r} - 1/\tilde{a})]^{1/2} \tag{18}$$

Hence, the pseudo velocity at $\tilde{r}$ is:

$$\tilde{v} = \mathcal{R}\begin{bmatrix} \Delta i \\ \Delta\Omega \\ \Delta\omega + \Delta\nu + \Delta\nu_{\text{bias}} \end{bmatrix} \frac{v}{|v|}\tilde{s} \tag{19}$$

Writing Eq. (3) in pseudo terms, and this can be done since the pseudo motion is Keplerian,

$$\tilde{v} = \tilde{g}^{-1}(r_T - \tilde{f}\tilde{r}) \tag{20}$$

Equating the right-hand sides of Eqs. (19) and (20),

$$\mathcal{R}v(\tilde{s}/\dot{s}) = \tilde{g}^{-1}(r_T - \tilde{f}\tilde{r}) \tag{21}$$

Solving for the actual velocity required to intercept r_T,

$$v_{\text{req}} = \mathcal{R}^T(\dot{s}/\tilde{s})\tilde{g}^{-1}(r_T - \tilde{f}\tilde{r}) \tag{22}$$

To recapitulate, the analysis was directed toward finding $\tilde{r}$ in order that Eq. (3) and the equations of Keplerian motion, which are easily computable, could be used to express v_{req} in the simple form of Eq. (22).

The orbital elements a and e can be selected on the basis of achieving a minimum energy arc,[2] or desired re-entry angle, or desired ballistic flight time. The orientation angles can be obtained[3] knowing r and r_T. Expressions for variations of the elements due to J2 are given by Geyling and Westerman.[4]

Example Test Cases and Numerical Results

The previous theory was programmed and the computational sequence is similar to that which a flight computer would require in an explicit guidance mode for upper-stage steering and engine cutoff.

An ephemeris generation program which employs numerical integration with a J2 field was modified to calculate the miss (minimum distance from r_T) of the simulated trajectory given the initial state r, v_{req} and the results are displayed in Table 1. Small miss is indicated for this ordered grid of target positions. Ignoring J2 in test case 7 causes a 24,400 ft miss; however, when including J2 effects the miss is only 235 ft.

Conclusions

An accurate analytic method has been developed to compute the velocity required for intercept for a space vehicle with perturbations present. Time of flight and perturbation effects involved are side calculations in this development; however, if they are not treated as separate calculations in the analysis, then for even short arcs the expressions become very complex.[5]

Most guidance schemes are tied to a reference trajectory which has been generated by numerical integration and hence can achieve mission changes only close to the reference. This development is independent of a reference trajectory which allows for a flexible respecification of mission. Also, it is efficient in terms of computation time because it does not involve numerical integration and this can be critical in in-flight operations.

Acknowledgment

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References

[1] White, B.D., "A Solution to 'The Velocity Required for Intercept' Problem for a Space Vehicle Including Perturbations," Master's Thesis, University of California, Los Angeles, March 1975.

[2] Battin, R.H., *Astronautical Guidance*, McGraw-Hill, New York, 1964.

[3] Bate, R.R., Mueller, D.D., and White, J.E., *Fundamentals of Astrodynamics*, Dover Publications, Inc., New York, 1971.

[4] Geyling, F.T. and Westerman, H.R., *Introduction to Orbital Mechanics*, Addison-Wesley, 1971.

[5] Robertson, W.M., *A Universal Solution to Lambert's Problem in the Presence of Arbitrary Perturbative Accelerations for Small Transfers*, Space Guidance Analysis Memo 6-70, M.I.T. Charles Stark Draper Laboratory, July 1970.

Announcement: 1978 Author and Subject Index

The indexes of the six AIAA archive journals (*AIAA Journal, Journal of Aircraft, Journal of Energy, Journal of Guidance and Control, Journal of Hydronautics, Journal of Spacecraft and Rockets*) will be combined and mailed separately early in 1979. In addition, papers appearing in volumes of the *Progress in Astronautics and Aeronautics* book series published in 1978 will be included. Librarians will receive one copy of the index for each subscription which they have. Any AIAA member who subscribes to one or more Journals will receive one index. Additional copies may be purchased by anyone, at $10 per copy, from the Circulation Department, AIAA, Room 730, 1290 Avenue of the Americas, New York, New York 10019. **Remittance must accompany the order.**

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